Pergamon

J. Appl. Maths Mechs, Vol. 62, No. 2, pp. 217–224, 1998

0021–8928/98/\$—see front matter

PII: S0021–8928(98)00028-8

VORTEX DAMPING OF SLOSHING IN TANKS WITH BAFFLES†

V. A. BUZHINSKII

Korolev, Moscow Region

(*Received* 18 *June* 1997)

The problem of damping the sloshing in tanks with sharp-edged baffles (thin inserts which partially span a longitudinal or transverse cross-section) is considered. Separation of the boundary layer and the formation of vortices occur at these sharp edges. It is assumed that the domains where there is significant vortex motion of the fluid are localized in small neighbourhoods of the sharp edges of the baffles. The non-linear vortex damping is determined from the distribution of the velocity intensity factors at these sharp edges in the same way as the linear damping, caused by the dissipation of energy in a boundary layer close to a wall, is determined from the fluid velocity distribution on the walls of a cavity. Both of the above-mentioned distributions are calculated by solving the same boundary-value problem on the oscillations of an ideal fluid. The second of the distributions characterizes the singular properties of the solutions of this problem on particular lines. A method based on the variation of the area of the baffles, which simplifies the calculation of the velocity intensity factors is described. The distinctive features arising when the method of finite elements is used are considered. The results of numerical calculations of the damping of sloshing in a cylindrical tank with a ring baffle are compared with experimental data.

1. Consider the sloshing of a fluid with a free surface in a tank containing structures with sharp edges. The surface tension of the fluid is neglected. The fluid is assumed to be incompressible and of low viscosity and the boundary layer is assumed to be thin. During the sloshing, the boundary layer becomes separated from the sharp edges of the structures and vortices form. We assume that the sloshing amplitudes are so small that the vortex patterns formed are localized in a small neighbourhood of the sharp edges of the baffles.

Outside small regions in the neighbourhood of the sharp edges and the boundary layer (on the walls of the tank and of the baffles) the motion of the fluid is assumed to be irrotational and is described by the displacement potential

$$\Phi = \Sigma s_n(t)\varphi_n(x,y,z) \tag{1.1}$$

in the form of a linear superpositioning of the natural modes of the oscillations φ_n $(n = 1, 2, \ldots)$, where x, y and z are the coordinates of a point in the fluid and t is the time. We know [3] that the natural modes of oscillation of a fluid are found as the solutions of the boundary-value problem

$$\Delta\varphi = 0,\ \ \partial\varphi/\partial\nu = 0 \ \text{ in } \ S; \partial\varphi/\partial\nu = \varkappa\varphi \ \text{ in } \ \Sigma \tag{1.2}$$

Here S is the surface of the walls of the tank and of the baffles wetted by the fluid, Σ is the free surface of the liquid, ν is the unit vector of the outward normal to these surfaces and $\varkappa = \omega^2/j$ is a frequency parameter (ω is the frequency of the oscillations of the fluid and j is the apparent acceleration). Taking the above assumptions into account, we refer the last boundary condition of (1.2) to the unperturbed plane free surface.

We now introduce the generalized coordinates s_n $(n = 1, 2, \ldots)$ into (1.1) using the condition

$$\max \partial\varphi_n/\partial\nu = \max \varkappa_n\varphi_n = 1 \ \text{ in } \ \Sigma \tag{1.3}$$

where $\varkappa_n = \omega^2/j$ is an eigenvalue of problem (1.2). They then determine the height of the waves on the free surface of the fluid. We note that only those modes of oscillation which bring about a displacement of the centre of mass of the fluid affect the motion of the tank.

Since vortex damping turns out to be non-linear, that is, it depends on the amplitude of the waves, we shall also consider other methods of selecting the generalized coordinates. We introduce the new coordinates r_n $(n = 1, 2, \ldots)$ by the formula

†*Prikl. Mat. Mekh.* Vol. 62, No. 2, pp. 234–242, 1998.

$$s_n = \frac{|\lambda_n|}{\mu_n} r_n \quad (\lambda_n = \rho \int_S \varphi_n \mathbf{v} dS, \ \mu_n = \rho \varkappa_n \int_\Sigma \varphi_n^2 dS) \tag{1.4}$$

where λ_n is the unnormalized hydrodynamic force vector, μ_n is the apparent mass coefficient, ρ is the density of the fluid, and the directions of the vectors λ_n and $\mathbf{r}_n$ are identical. These generalized coordinates determine the displacements of the masses $m_n = \lambda_n^2/\mu_n$ ($n = 1, 2, \ldots$) of the mechanical analogue [4]. The angles of deflection of the equivalent pendulums

$$\alpha_n = \varkappa_n r_n \tag{1.5}$$

are often chosen as generalized coordinates.

The coordinates $r_n(t)$ and $\alpha_n(t)$ ($n = 1, 2, \ldots$) are convenient since they are independent of the normalization of the natural modes of oscillation.

As a consequence of the above assumptions, in the zeroth approximation we determine all the hydrodynamic characteristics, apart from the dissipative characteristics, within the framework of the concept of the irrotational motion of an ideal fluid. Although, in irrotational motion, the fluid velocity becomes infinite at the sharp edges of the baffles, no such singularities arise when calculating these characteristics [4, 5]. Experimental data [4, 6] confirm the admissibility of such an approach in the case of the lowest slosh modes with $s_n(t)/D < 0, 1$ ($n = 1, 2$), where D is the maximum linear dimension of the free surface of the fluid.

2. We will now determine the damping and estimate of other corrections due to the vortex motion of the fluid in small domains. In order to do this, we consider the steady forced oscillations of the fluid at a frequency ω which is close to one of the natural frequencies ω_n, assuming that they are maintained by some external source of excitation. Neglecting, under these resonance conditions, the effect of the other slosh modes, we write the potential (1.1), omitting the subscript n for the number of the slosh mode, in the form

$$\Phi = s\varphi(x, y, z) \sin \omega t \tag{2.1}$$

where s is the constant amplitude.

We now distinguish three characteristic domains in the bulk of the liquid

1. the domain of irrotational motion of the fluid;

2. the domain of a small "distant" neighbourhood of the sharp edges where the fluid flow is described by the principal singular term of the solution of the problem of irrotational motion in which the velocities on the sharp edges become infinite;

3. the domain of a small neighbourhood which is closer to the sharp edges where the fluid flow is substantially a vortex flow and there is a periodic change on the vortex patterns as a result of boundary layer separation.

We determine the ratios of the orders for the dimensions of these domains in the following way: the characteristic size of the baffles and the tank are much greater and the boundary-layer thickness is much smaller than the characteristic size of the domain where there is significant vortex motion of the fluid.

Under the assumptions which have been made, the formation of vortex patterns in the neighbourhoods next to the sharp edges is completely determined by the irrotational flow of the fluid in the small "distant" neighbourhoods of the edges. The domain of the distant neighbourhoods of the sharp edges are immediately adjacent to their closest neighbourhoods so that the characteristic dimension of its transverse cross-section is also much less than the width of the baffles. Therefore, the irrotational flow in this domain is close to plane flow and, in the cylindrical system of coordinates $xr\theta$, the x axis of which is directed along the tangent to the line of the sharp edge and the angle θ is measured from the surface of the baffle, the potential (2.1) and the velocities v_r, v_θ can be written in the form

$$\Phi = \left[K_v \sqrt{\frac{2r}{\pi}} \cos\frac{\theta}{2} + O(r)\right] \sin \omega t \tag{2.2}$$

$$v_r = \left[\frac{K_v}{\sqrt{2\pi r}} \cos\frac{\theta}{2} + O(1)\right] \sin \omega t, \quad v_\theta = \left[-\frac{K_v}{\sqrt{2\pi r}} \sin\frac{\theta}{2} + O(1)\right] \sin \omega t$$

where K_v is the velocity intensity factor (VIF), which we note depends linearly on the amplitude s. The following terms in the asymptotic expansion (2.2) may also depend on the radii of curvature of the baffle if it is not plane and on the radius of curvature of the line of the sharp edge.

The fluid flow in the distant neighbourhood of the edges is therefore characterized by the single parameter K_υ, which can change along the length of the contour l of the sharp edge. In this case, the expenditure of energy in forming vortices during the period of the oscillations is determined using the formula [1]

$$\Delta E = B\rho\omega^{-2/3}\int_l K_\upsilon^{8/3} dl \tag{2.3}$$

where $B \approx 2$ is a universal constant.

In the process under consideration, the damping of the oscillations of the fluid is associated with the transfer of energy from an irrotational form of motion to a vortex form of motion and is independent of the viscosity of the fluid. This is confirmed by experimental data [4, 6] over a wide range of high Reynolds numbers, calculated from the frequency of the oscillations and the characteristic dimension of the tank. The secondary process of the energy dissipation in the very small-scale vortices of the vortex patterns which are formed occurs as a consequence of the viscosity.

The energy of the irrotational flow of the fluid can be written in the form $E = \mu\upsilon_0^2/2$, where μ is the apparent mass (1.4) and $\upsilon_0 = \omega s$ is the characteristic velocity of the fluid for the corresponding mode of the oscillations. For the damping coefficient (DC), we find

$$\delta = \frac{\Delta E}{2E} = K\left(\frac{\upsilon_0}{R\omega}\right)^{2/3}, \quad K = B\frac{\rho R^3}{\mu}\int_l \left(\frac{K_\upsilon^2}{R\upsilon_0^2}\right)^{4/3} d\frac{l}{R} \tag{2.4}$$

where R is the characteristic dimension of the tank. In the case of weak damping, the DC, δ, corresponds to the log decrement of the oscillations. Since $\upsilon_0 = \omega s$, it follows from (2.4) that the DC depends on the relative amplitude of the wave s/R in the free surface of the fluid to the power of 2/3. On taking (1.4) and (1.5) into account, the DC can be represented as a function of the value of the amplitude of the generalized coordinate r_n or α_n.

Relation (2.4) only gives the non-linear vortex part of the DC for the oscillations of the fluid. The linear part of the DC, which is associated with the dissipation of energy in the boundary layer, can be independently determined using the formula [7]

$$\delta = \frac{\pi}{\sqrt{2}}\frac{\rho R^3}{\mu}\mathrm{Re}^{-1/2}\int_S (\nabla\varphi)^2 \frac{dS}{R^2}, \quad \mathrm{Re} = \frac{\omega R^2}{\nu} \tag{2.5}$$

where ν is the kinematic viscosity of the fluid and Re is a dimensionless parameter, equivalent to the Reynolds number. When account is taken of the fact that the characteristic size of the domain of vortex motion $d \approx (K_\upsilon/\omega)^{2/3}$ [1], in the case of the integration in (2.5) it is necessary to exclude from both sides of the baffles the area which is formed by the contour of the sharp edge l and the contour spaced at a distance d from it.

We will now consider the effect of vortex patterns in the neighbourhood of the sharp edge s on the apparent mass (AM) of the fluid. In order to do this, we represent the AM of any slosh mode in the form $\mu = \mu_0 + \Delta\mu$, where μ_0 is its magnitude for infinitesimal amplitudes of the oscillations. Under the above assumptions, the correction $\Delta\mu$ can only depend on ρ, ω and K_υ. Invoking similarity arguments [8], we find that

$$d\Delta\mu / dl = B_\mu \rho\omega^{-4/3} K_\upsilon^{4/3} \tag{2.6}$$

where $B_\mu \approx 0.5$ is a universal constant, an approximate value of which can be determined with an error of $\pm 10\%$ from experimental data [6, 9] on the oscillations of plates in a fluid.

By integrating (2.6), we represent the correction to the AM in the form

$$\frac{\Delta\mu}{\mu_0} = K_\mu\left(\frac{\upsilon_0}{R\omega}\right)^{4/3}, \quad K_\mu = B_\mu \frac{\rho R^3}{\mu_0}\int_l \left(\frac{K_\upsilon^2}{R\upsilon_0^2}\right)^{2/3} d\frac{l}{R} \tag{2.7}$$

In the case of a plate of infinite span with a width R executing oscillations perpendicular to its plane in an unbounded liquid, $\mu_0 = \pi\rho R^2/4$ and $K_\upsilon^2 = \pi R\upsilon_0^2/2$ and, hence, from (2.7), we find the relation

$$c_m = \mu/\mu_0 = 1 + 1.72(\upsilon_0/(R\omega))^{4/3}$$

for the coefficient of apparent mass.

A comparison of this relation with experimental data [6, 9] (the small open circles) is shown in Fig. 1, plotted using the axes $X = \upsilon_0/(\omega R)$ and $Y = c_m$. In spite of its asymptotic nature, there is satisfactory agreement with the experimental data up to amplitudes comparable with the plate width.

It can be seen from (2.7) that the correction to the AM depends on the relative slosh amplitude $\upsilon_0/(\omega R) = s/R$ to the power of 4/3 and it is therefore a quantity of the next order of smallness compared with the vortex damping (2.4). Consequently, the change in the AM as a consequence of the vortex motion of the fluid can be neglected to a first approximation over the range of small amplitudes of oscillation mentioned above. In the case of tanks with a characteristic dimension $R \sim 1$, the vortex damping (2.4) can be two orders of magnitude greater than the linear damping (2.5) over the same range of amplitudes of the oscillations. The change in the AM due to the boundary layer is proportional to $\mathrm{Re}^{-1/2}$ and is very small [7].

Comparing expressions (2.4) and (2.5), we see that, in order to determine the linear damping, it is necessary to know the distribution of the velocities **v** of the fluid on the surface of the walls of the tank and of the baffles while, in order to determine the non-linear vortex damping, it is necessary to know the velocity intensity factors (VIFs), K_υ, on the lines of the sharp edges of the baffles. These distributions are found by solving the same boundary-value problem (1.2) and, in this sense, the hydromechanical models used to determine the linear and vortex damping are approximations of the same order. However, it is much more difficult to calculate the VIFs since they characterize the singular properties of the solutions of problem (1.2) on particular lines. The usual numerical methods are therefore not suitable for this. We note that an analogous mathematical problem arises in linear fracture mechanics when calculating the stress intensity factors on the sharp edges of cracks in a solid [10].

3. We will now describe a method of determining the VIF, K_υ, which is based on a variation of the area of the surface of the baffles. We conceptually reduce the baffle area by a small amount

$$\delta S = \int_l \delta n(l)\,dl$$

where δn is a variation of the normal to the contour of a sharp edge in a plane tangential to it. In this case, the eigenfunctions and eigenvalues of problem (1.2) change and we therefore denote them by φ_n^δ and $\varkappa_n^\delta$ $(n = 1, 2, \ldots)$. To simplify the subsequent account, we shall omit the subscript n, as was done above when considering one of the slosh modes.

The functions φ and φ^δ, that is, φ_n and φ_n^δ, are harmonic inside the initial boundary surface, and Green's formula

$$\int_{\Sigma+S-\delta S} \varphi\frac{\partial\varphi^\delta}{\partial\nu}dS + \int_{\delta S}\varphi\frac{\partial\varphi^\delta}{\partial\nu}dS = \int_{\Sigma+S}\varphi^\delta\frac{\partial\varphi}{\partial\nu}dS \tag{3.1}$$

can therefore be used, having separated out integration with respect to a variation in the area δS on the left-hand side. Note that the integration must be carried out over both sides of the baffle.

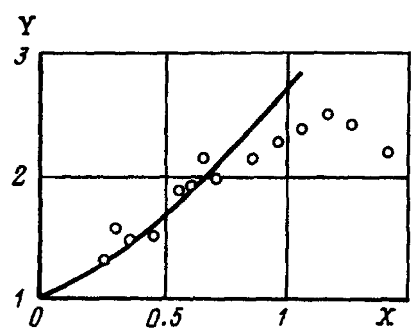

Fig. 1.

From (3.1), taking account of the boundary conditions of problem (1.2) and the asymptotic relations (2.2) for the potential φ an the velocity $\partial\varphi^\delta/\partial\nu$ in the neighbourhood of the sharp edges, we obtain

$$\int_l K_\upsilon^2 \delta n(l) dl = -(\varkappa - \varkappa^\delta) \int_\Sigma \varphi\varphi^\delta dS \tag{3.2}$$

It is obvious that φ^δ is only slightly different from φ, and we therefore write $\varphi^\delta = \varphi + \delta\varphi$. Using the notation $\delta\varkappa = \varkappa - \varkappa^\delta$ and neglecting $\varphi^\delta\varphi$, on the right-hand side of (3.2) under the integral sign, compared with φ^2, we finally obtain

$$\int_l K_\upsilon^2 \delta n(l) dl = -\delta\varkappa \int_\Sigma \varphi^2 dS \tag{3.3}$$

Relation (3.3) is satisfied for any slosh mode.

By taking $\delta n(l) = \delta n = \text{const}$ along the contour of the sharp edge and using (1.4), we can represent (3.3) in the dimensionless form

$$\int_l \frac{K_\upsilon^2}{R\upsilon_0^2} \frac{dl}{R} = -\frac{\delta\varkappa}{\varkappa\delta n / R} \frac{\mu}{\rho R^3} \tag{3.4}$$

where, as above, $\upsilon_0 = \omega s$ is the characteristic velocity, μ is the apparent mass and, if n is the number of the slosh mode, then $\omega = \omega_n$, $\mu = \mu_n = \varkappa_n$, $\delta\varkappa = \delta\varkappa_n$ and $s = s_n$ (the amplitude of the wave on the free surface). Each mode has its own dimensionless value of the square of the VIF $K_\upsilon^2/(R\upsilon_0^2)$ on the contour l of the sharp edge.

In certain cases, the use of relations (3.3) and (3.4) enables us to simplify the determination of the VIF substantially and, in other cases, to control the accuracy of the calculation. For this purpose it is necessary to solve problem (1.2) several times for different small values of δn.

If the surface of a baffle in a small segment of the contour l is changed by δS and it is assumed that $K_\upsilon = \text{const}$ in this segment, it then follows from (3.3) that

$$K_\upsilon^2 \approx -\frac{\delta\varkappa}{\delta S} \int_\Sigma \varphi^2 dS \tag{3.5}$$

or, in dimensionless form

$$\frac{K_\upsilon^2}{R\upsilon_0^2} \approx -\frac{\delta\varkappa}{\varkappa\delta S / R^2} \frac{\mu}{\rho R^3} \tag{3.6}$$

Relation (3.5) or (3.6) can be used for the direct calculation of the VIF when solving three-dimensional problems using finite element methods, but, in this case, eigenvalue problem (1.2) has to be solved a large number of times. A favourable exception is the two-dimensional problem when $K_\upsilon = \text{const}$ over the whole length of the sharp edge. Two-dimensional flow conditions are well satisfied in the case of the transverse oscillations of a liquid in a long horizontally arranged cistern with longitudinal damping baffles of constant width. In this case, problem (1.2) has to be solved twice in order to determine the VIF for a single baffle using (3.5) or (3.6).

4. We will now describe the use of finite element methods to solve a class of problems which is of practical importance. We consider the sloshing of a fluid in a tank, the surface of which is a surface of rotation containing transverse baffles which preserve the axial symmetry of its cavity. These may be annular, conical or cylindrical baffles or baffles made up of different combinations of these.

Assuming that the apparent acceleration vector, $\mathbf{j}$, is directed along the longitudinal axis of the tank in the opposite direction to the x axis of the cylindrical system of coordinates $xr\theta$, we will seek the potential in the form $\varphi = f(x, r)\cos\theta$, which is confined to just those modes of oscillation for which the principal vector of the hydrodynamic forces is non-zero. Since the angular coordinate θ is separated out, we find the solutions of problem (1.2) among the functions for which the discrete functional

$$I(f)=\Sigma\int_{S_i}\left[\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial r}\right)^2+\frac{f^2}{r^2}\right]rdrdx-\varkappa\Sigma\int_{l_k}f^2rdl \tag{4.1}$$

is an extremum. Here S_i is the area of the ith finite element in the domain of the radial cross-section of the fluid volume of the cavity, l_k is the kth side of one of the finite elements belonging to the line of the free surface of the fluid in this cross-section, ΣS_i is the cross-section area and Σl_k is the line of the free surface in this cross-section.

We will use isoparametric triangular finite elements of quadratic order with simplex coordinates [11, 12]. We shift the middle points of the sides of the finite elements, which meet in the sharp edge, by 1/4 of the length of the corresponding side towards the sharp edge. In this case, a Jacobian transformation from the x, r coordinates to the simplex coordinates ensures the required singularity [13] in the fluid velocities close to the sharp edge which considerably increases the accuracy of the solution of the problem. The one- and two-dimensional integrals in (4.1) are calculated using the Gauss quadrature formulae with 3 and 4 mesh points, respectively.

It is convenient to consider the baffles as being infinitesimally thin. Therefore, the nodal points, lying on the different sides of the baffles in the lines corresponding to these baffles in the domain of the radial cross-section, are distinguished but, in this case, they have the same coordinates (double nodal points).

The condition for an extremum of the discrete functional (4.1) leads to a common eigenvalue problem in linear algebra in which the values of the functions f at the nodal points of the finite elements are unknown. The problem is solved by the method of iterations in the subspace, assuming that one of the matrices has a band structure and that the other is very sparse matrix.

In the case under consideration, the lines of the sharp edges of the baffles are circles. Along these lines, the dependence of the VIF on the angle θ is obviously the same as for the potential φ and, therefore

$$K_\upsilon(\theta)=K_{\upsilon 0}\cos\theta \tag{4.2}$$

Substituting (4.2) into (3.4), we find that

$$\frac{\pi r_0}{R}\frac{K_{\upsilon 0}^2}{Rv_0^2}=-\frac{\delta\varkappa}{\varkappa\delta n/R}\frac{\mu}{\rho R^3} \tag{4.3}$$

where r_0 is the radius of the line of the sharp edge and $K_{\upsilon 0}$ is the maximum value of the VIF on the line of this edge.

By first calculating the eigenvalues $\varkappa=\varkappa_n$ $(n=1,2,\ldots)$ and the apparent masses $\mu=\mu_n$ $(n=1,2,\ldots)$ when $\delta_n=0$ using finite element methods and then the changes in the eigenvalues $\delta\varkappa=\delta\varkappa_n$ $(n=1,2,\ldots)$, the VIF, $K_{\upsilon 0}$, corresponding to these modes are determined for a given baffle using formula (4.3). After this, taking account of (4.2), the contribution from a given baffle to the damping coefficient of the oscillations of the fluid $\delta=\delta_n$ $(n=1,2,\ldots)$ is determined using formulae (2.4). The overall damping coefficients are found by summing the contributions from all of the baffles. As a rule, it is only the first slosh mode $(n=1)$ which is of practical interest.

The algorithm which has been described was programmed in FORTRAN and included in a general program designed to determine the hydrodynamic characteristics of fuel tanks.

5. We will now present some results of calculations using finite element methods of the damping of the fundamental first mode of the oscillations of a fluid in a cylindrical tank with a single ring baffle. It is assumed that the apparent acceleration vector is directed along the tank axis and is perpendicular to the plane of the baffle. Suppose b is the width of the baffle, h is the height of the fluid above its plane and that Δ is the gap between the baffle and the cylindrical wall of the tank. The radius of the cylinder R is taken as the characteristic dimension.

Figure 2 shows the results of the finite element calculation of the VIF ($Y=K_\upsilon^2/(R\upsilon_0^2)$) and of the coefficient K in formula (2.4) which determines the magnitude of the damping of the sloshing as a function of the width b of a baffle, set up without a gap ($\Delta=0$) and sunk to depths of $h/R=0.2$ and $h/R=0.3$. The smaller values of the VIFs and K correspond to the greater depth. The values of K are shown by the solid lines obtained by interpolation using the results of the calculations, while the VIFs are shown by the dashed lines. Maximum damping is obtained for $b/R\approx 0.4$.

Relation (2.4) is compared with the experimental data in Fig. 3 for the following versions

Version	1	2	3	4
b/R	0.1	0.1	0.2	0.2
h/R	0.3	0.2	0.3	0.2

The results of the calculations are represented by the solid lines; the results of a treatment of the experimental data [4] using an empirical dependence of the form $\delta = K\sqrt{s}$ [14] are represented by the dashed lines and the experimental data obtained by Mel'nikova after the publication of the book [4] are represented by the points.

The results of the investigation into the effect of a gap between the baffle and the tank wall are shown in Fig. 4. They were obtained in the case of ring baffles of constant width $b/R = 0.12$ arranged at a depth $h/R = 0.1$ from the free surface of the liquid. Small gap values, for which an increase in the damping compared with baffles set up without a gap has been found experimentally, are of interest.

The dependences of the VIF $K_{\upsilon 0}$ on the size of the gap Δ plotted using the axes $X = \Delta/b$, $Y = K_{\upsilon 0}^2/(R\upsilon_0^2)$ are represented by the solid lines. Large values of VIF correspond to an external edge located close to the wall. As the gap is reduced, the VIFs on both sharp edges increase. In this case, the VIF on the inner edge tends to its own limiting value for a baffle without a gap and it makes sense to calculate the VIF on the outer edge in accordance with one of the assumptions made until $\Delta \gg \sqrt{(\nu/\omega)}$, that is, while the gap width is much greater than the boundary-layer thickness.

The dependence of the coefficient K on the gap width ($X = \Delta/b$), constructed from formula (2.4) using the VIF values depicted by the solid lines, is shown by the dashed line. All the lines in Fig. 4 are interpolations using the results of the calculations. It can be seen that an increase in the damping is actually observed when $\Delta/b < 0.12$. These results are in qualitative agreement with the experimental data obtained in the case of other baffles. The position and value of the maximum in the dashed line depends on the viscosity of the fluid which, as noted above, restricts the growth of the VIF on the outer edge of the baffle. Note that, when $\Delta/b = 0.04$, the gap is less than 1/200 of the radius of the tank, and the proposed approach therefore enables one to determine the non-linear vortex damping of the sloshing taking account of extremely small-scale constructional features.

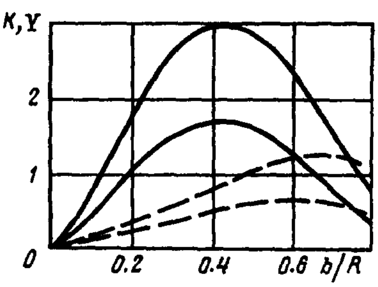

Fig. 2.

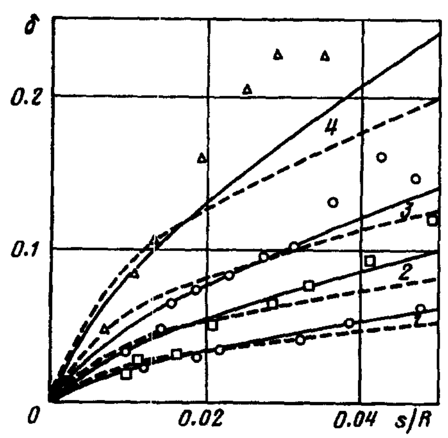

Fig. 3.

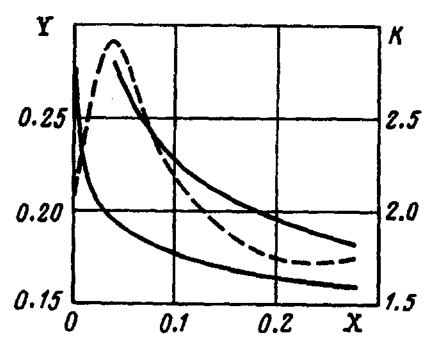

Fig. 4.

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Translated by E.L.S.